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Supreme Quarto

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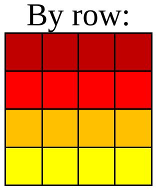
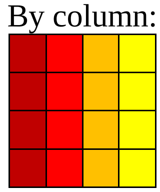
Quarto is a board game that consists of a playing board divided into four rows and four columns of squares, creating 16 squares, and 16 game pieces each with four distinctive characteristics based on four binary categories: height, color, shape, and indentation. A quarto occurs when four pieces in a row, column, or diagonal share a characteristic on the playing board.

I claim that there are 12288 configurations to obtain a supreme quarto. To explain the definition of a supreme quarto, we will first observe a regular quarto:

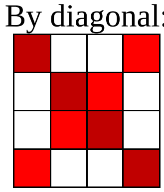
By row:



By column:



By diagonal:



Each color represents a quarto. We can clearly see that there are 10 possible ways to obtain a quarto. A supreme quarto occurs when all 10 quartos occur at once.

Because each characteristic of a piece is binary, we can actually assign each characteristic to a zero or one, and since there are four characteristics, this will give us a set of four digits for each of the 16 pieces ranging from 0000 to 1111. We can arrange these numbers accordingly on the playing board as follows:

x	3,4	3,4	3,4	3,4	x
1,2	0000	0001	0010	0011	
1,2	0100	0101	0110	0111	
1,2	1000	1001	1010	1011	
1,2	1100	1101	1110	1111	

The number(s) in the bottom four cells of the far left column indicate which position in the each of the four-digit sets of that row has the same digit. Likewise, the number(s) in the middle four cells of the absolute top row indicate which position in the each of the four-digit sets of that column has the same digit. The number(s) (in this case “x”) in the absolute top-left and top-right cells indicate which position in the each of the four-digit sets of that diagonal has the same digit. We will call these bordering numbers. As we can see, this configuration does not contain a supreme quarto, although it does contain eight regular quartos (ignoring multiple quartos per row and column). We can rearrange this configuration to form a new configuration that does contain a supreme quarto:

2	3	3	3	3	4
1	0111	0010	0100	0001	
1	0011	0110	0101	0000	
1	1110	1011	1101	1000	
1	1111	1010	1001	1100	

We can now say that there exists at least one configuration that forms a supreme quarto. Let's divide this configuration into four two by two partitions:

2	3	3	3	3	4
1	0111	0010	0100	0001	
1	0011	0110	0101	0000	
1	1110	1011	1101	1000	
1	1111	1010	1001	1100	

Obviously, interchanging the diagonals of each partition that would form a regular diagonal quarto will not affect the bordering numbers indicating diagonals. As for each partition's counter-diagonal, each of the digits in the four-digit sets indicated by both the row and column bordering numbers are the same. Hence, each of the four-digit sets in each of the partition's diagonals can be interchanged without affecting the bordering number, and therefore, creating a new configuration that forms a supreme quarto. Since there are two diagonals per partition, this means that each partition has four configurations. If we combine all four partitions, we will have 4^4, or 256, configurations that will not affect the bordering numbers.

If we now look at each partition as a whole, based on the same principles described previously regarding the bordering numbers not being affected, a supreme quarto can be formed by interchanging each partition with its corresponding diagonal partition. However, if two partitions are interchanged, the counter-diagonal partitions must be interchanged as well in order to obey the same principle. Since there are only two configurations using this technique, we can now multiply our previous number 256 by 2 to obtain 512 absolute configurations that will not affect the bordering numbers.

Since each position in the four-digit set is identical, meaning each will hold either a zero or one, then the bordering numbers can be rearranged in any order. If we read each configuration of bordering numbers as: *rows, top-left to bottom-right diagonal, columns, bottom-left to top-right diagonal*, the configuration above would be read as: 1, 2, 3, 4. The number of bordering configurations that can be arranged is simply 4!, or 24.

When we multiply the number of configurations of bordering numbers by the number of configurations of the four-digit sets that will not affect a single configuration of the bordering numbers, we will obtain the total number of configurations of the four-digit sets that will form a supreme quarto:

$$4! \cdot 4^4 \cdot 2 = 24 \cdot 256 \cdot 2 = 12288$$

We know there is at least this many configurations to form a supreme quarto. Notice that all the rows and columns have the same bordering number and the diagonals have different bordering numbers. If we attempt to have a row have a different bordering number than the rest as follows:

x	3	3	3	3	4
1	0111	0110	0100	0101	
2	0011	0010	0001	0000	
1	1110	1011	1101	1000	
1	1111	1010	1001	1100	

The configuration will no longer be a supreme quarto as we had to sacrifice a diagonal. This would apply to any of the eight four-digit sets that would form the regular quarto. One could try interchanging a row with a column:

2	x	x	x	x	x
1	0111	0010	0100	0001	
3	0011	0110	1010	1110	
x	0101	1011	1101	1000	
x	1111	0000	1001	1100	

This configuration is worse as it removes several regular quartos. We could attempt to have the diagonals be the same bordering number:

2	x	3	3	x	2
1	0111	0010	0100	0001	
x	1011	0110	0101	0000	
x	1110	0011	1101	1111	
1	1000	1010	1001	1100	

This also proves impossible as when we move any four-digit set to obtain the quarto we wish, another quarto gets removed. Thus, there can be no other configurations for a supreme quarto. In conclusion, there are only 12288 configurations to form a supreme quarto.

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